**Physics 225** **COMPUTATIONAL MECHANICS** **Winter Term, 2009**

**INSTRUCTOR:** Megan K. Pickett, Youngchild 105, X6269, `megan.pickett@lawrence.edu`
**CONSULTANT:** Julia Ziege
**CLASS HOURS:** MWF 9:50-11:00 AM, Y-115 (or Y-138, see below)
**CLASS WEBSITE:** `http://www.lawrence.edu/fast/pickettm/p225w09/`

**OFFICE HOURS:** 11:30 am-12:30pm MW; 2-4 pm TR; other times by appointment
**CONSULTING SCHEDULE:** Very soon after the start of the term, an experienced and talented student consultant (our very own Julia Ziege) will be on duty in the CPL to provide assistance with the computer portions of Physics 225. Once they have been determined, times will be announced in class and posted on the bulletin board next to the door into the CPL.

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**Welcome to Computational Mechanics, Physics 225!** This term, we will tackle a variety of topics in mechanics, including forces & potentials, kinematics & dynamics, rotational physics, orbits, and oscillations. Physics 225 is also known, of course, for introducing students to a suite of numerical techniques and software packages, especially IDL and MAPLE. Computational Mechanics has a well-deserved reputation as a challenging and rewarding class, so it is important that you keep up with the reading and the work, and why you have been given a detailed schedule for the term. If you have any question, concern or problem, please do not hesitate to contact me using any means you find helpful and convenient (I have yet to receive a message by sema-fore, but if that's your thing, I'll be happy to learn it. We're a full service department here at Lawrence.)

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**REQUIRED TEXTS:** The following three items are required for this course and will be distributed at the first class. Two or three weeks into the term, I will instruct the Business Office to charge your student account $51.35 for these materials. I will, of course, have also to add the 5% ($2.57) Wisconsin sales tax. The total charge posted to each of your accounts will be $53.92.

- *Notes for Computational Mechanics* (*NCM*) by David M. Cook; $14.10. You will have the edition dated 15 December 2008 (202 + xiv pages). This locally produced manuscript is the primary text for the mechanics taught in this course. The alternative would be a formally published text, which would cover more mechanics than we will cover and would be *much* more expensive. For your reference, some of the possible texts that *might* have been chosen are made available on reserve in the library. (See the statement about the reserve shelf on page 2 in this fact sheet.) We will definitely cover all the material in Chapters 1-5.[1]
- *Computation and Problem Solving in Undergraduate Physics* (*CPSUP*) by David M. Cook; $30.25. You should have the edition dated 6 December 2007 (500 + xx pages) and a listing of a few edits or the edition dated 10 December 2008.[2] The edition contains all components

[1] An additional section on an advanced topic (Lagrangian Mechanics) is included in case we have time at the end of the term; those of you who take Advanced Mechanics will cover this lovely technique for solving dynamics problems.

[2] The edits are printed on a single side of the page so you can make the larger of the edits by clipping the new text and taping it into your book. You should make all the edits in your copy of the book. Note, however, that I have included in the assignments an explicit pointer to an edit when that edit bears on the assignment. The edits are already incorporated in the 2008 edition of *CPSUP*

of *CPSUP* that are pertinent to Lawrence. We will, however, make use of only a portion of the book. I am providing all of it on the confident assumption that the rest will be valuable to you as you continue your studies of physics at Lawrence. I believe this book is one you will want to make use of throughout your time at Lawrence (and perhaps beyond). You will first need *CPSUP* when you start Assignment 2.

- *Local Guide* (*LG*) to accompany *CPSUP* (Edition dated 14 December 2007; 69 pages); $7.00.

**RECOMMENDED SUPPLEMENTARY TEXT:** R. Shankar, *Basic Training in Mathematics: A Fitness Program for Science Students* (Plenum Press, New York, 1995; ISBN 0-306-45036-4 (paper)). Taking a physicist's perspective, this book reviews a lot of mathematics, most of which you already know at some level. The Department has agreed that the book is a worthwhile addition to your personal library. Assignments in Physics 225 will occasionally direct your attention to portions of the book as supplementary reading. The book will also be referred to in subsequent physics courses. I think it would be worthwhile for you pick the book up and read a few pages every once in awhile, simply to help you develop increased familiarity with the mathematics pertinent to physics. There is (supposed to be) one copy of the book in the CPL. There is a second copy on the Physics 225 reserve shelf at the main library.

**LIBRARY RESERVE:** I will place several books on reserve for your use in the main library. Copies of *some* of these materials are also available in the library in the CPL. In addition, vendor manuals for the software packages we will be using are on the shelves in the CPL. (See the bibliographies at the ends of chapters in *CPSUP*.) You may wish to read portions of these materials to supplement your study of *LG*, *NCM*, and *CPSUP*. Possibly useful supplementary materials include:

- Barger and Olsson, *Classical Mechanics: A Modern Perspective*
- Marion, *Classical Dynamics*
- Slater and Frank, *Mechanics*
- Goldstein, *Classical Mechanics*
- Feynman, *Lectures in Physics, Volumes I and II*
- Symon, *Mechanics*
- Shankar, *Basic Training in Mathematics*

**CONFERENCE SESSIONS:** Four conference sessions are scheduled at intervals through the term. These sessions provide opportunity for detailed discussion of the current material. In addition, in each session, about three students will be asked to present solutions to designated problems; non-presenting members of the class are expected to ask questions of clarification or expansion when appropriate. Part of your course grade will be comprised of your performance during these sessions (see below). *Attendance is mandatory and will be checked.* I encourage you to see me *before* your time in order to go over your presentation with me. Some new material may be presented in these sessions if we are significantly enough behind the planned schedule.

**ASSIGNMENTS:** Seven problem assignments, all of which are described later in this syllabus, are distributed during the term. Each assignment identifies readings in *NCM*, *CPSUP*, and *LG*, occasionally suggests readings in alternative texts (including Shankar), and—most important—designates several problems that you should solve as a test that you understand the material covered by the assignment. Problems are divided into two groups. You should make a serious effort to solve *all* of the problems in Group 1 and ask about any that you are unable to solve. Think of problems in Group 2 as interesting diversions to be attacked only after you have mastered the remainder of the assignment–or to be used a further study aids for the exams. Many of the problems presented at the conferences will come from these problems.

The problems in Group 1 are to be written up *carefully*, *thoroughly*, and *neatly*. After the class in which an assignment is due, I will place solutions for the problems in Group 1 (and perhaps some of the problems in Group 2) in a notebook that will live in Y-104/CPL.

*NO LATE ASSIGNMENTS WILL BE ACCEPTED WITHOUT SPEAKING TO ME PRIOR TO THE DUE DATE. Extensions may be given if an acceptable and verifiable excuse is presented to me beforehand.*

*MY HOMEWORK DEAL FOR YOU:* If you make an *honest attempt* to complete *every* required problem AND turn in *every* problem set, I will drop your lowest homework score when computing your final average. How cool is that? Let me repeat, however: you MUST do every single assigned problem, whether or not you get the right answer, to be eligible for this reward. I, of course, am the final arbiter of what constitutes an honest attempt, but suffice it to say that merely writing $F = ma$ and leaving a problem at that will NOT count. The goal–my goal–is to get you in the habit of doing all the problems as preparation for understanding and the exams.

As is true with all the faculty in the physics department, please recognize that I believe a solution to a problem is FAR more important than a mere answer–that the method by which you solve a problem is often more important than the end result. One of my goals for this course is for you to develop *physical intuition*: the ability to quickly recognize when the answer to a problem–or the method with which you are attempting to derive an answer–makes sense physically. This is a hard talent to come by, and yet it is one of the more useful skills you will develop as a student in this course and beyond. So, when you come to the end of a problem, ask yourself: are the units correct? does the magnitude of the answer make sense–is the answer even in the ballpark? was the method you used appropriate for the problem?

To help you with this, I require that all submitted solutions must be legible and *complete*, i.e., they must contain not only the answer, not only a succession of equations, but also enough narrative to make clear how you thought your way carefully and logically from first principles. *THIS IS THE ONLY WAY TO RECEIVE FULL CREDIT ON A GIVEN PROBLEM. YOU CAN GET THE "RIGHT" ANSWER AND STILL LOSE POINTS IF YOU ARE NOT EXPLICIT, DETAILED, AND CLEAR.* It is your job in your submitted work to convince me that you have identified the applicable principles, recognized clearly what those principles are, and worked your way step-by-step from that starting point to the solution. Pay attention not only to the substance of your solution but also to its exposition. Solutions that look like your first draft on a sheet of scrap paper are not acceptable. Work that is indecipherable will receive no credit, and, further, you may be required to word-process future assignments.

Note as well that, when you include computer-produced graphs in a solution, your solution will be *incomplete* unless the accompanying narrative describes how you produced the graphs and indicates the specific statements you submitted to whatever program you used. All graphs should include titles, appropriate labels, and a short caption explaining the figure. You WILL lose points if you do not correctly annotate your figures.

**COMPUTER:** Recognizing the growing importance of skills in using assorted computational resources both to the upper-level physics curriculum at Lawrence and to the contemporary practice of physics, engineering, and other sciences, the Department of Physics introduced Physics 225 (*Computational Mechanics*) in the winter of 2003. Physics 225 replaces the former Physics 210 (*Mechanics*). The primary objective of this change is to incorporate a firm and universal introduction to computational approaches to problems in physics. Coverage of some of the mechanics (general rigid body motion, Lagrangian and Hamiltonian mechanics, collisions and scattering, ...) previously treated in Physics 210 have been curtailed or deleted altogether to make room for the addition of topics in graphical visualization, numerical solution of ordinary differential equations, and numerical evaluation of integrals.[3]

On several occasions during the term, class sessions will provide orientation and assignments will expect you to work through computer-based tutorials and make use of computer-based approaches to assigned problems. As you learn about the capabilities of IDL and MAPLE, please feel free to use these resources not only to address exercises that direct you specifically to the computer but also whenever else it seems appropriate to you to exploit our computational resources. The more you practice, the more fluent and comfortable you will become.

Note particularly exercise SP1.1 on the *first* assignment, in which you are asked to familiarize yourself with the workstations, set up your account, and explore one or two simple applications. There is nothing to be handed in when you finish SP1.1. Nonetheless, *please make it a point to do that exercise as soon as possible.* The *second* assignment will expect you to have developed the beginning skills that are the aim of SP1.1. *Please note also that there are more students than there are workstations; you cannot* all *wait until the last minute to start this–or any–exercise.*

**THE MIGHTY BIG PROBLEM:** The last week of the course is reserved for a final problem of your own construction. I will hand out materials regarding this final project after the midterm. The purpose of the MBP is to challenge you to apply the lessons from the term in as broad and innovative manner as possible. The write-up, which must be word-processed, will be due on the saturday after the last day of classes. Part of your grade for the MBP will be based on a 10 minute presentation of your problem and solution.

**EXAMINATIONS:** There will be a single hour examination, and a final examination in this course. All examinations are closed-book; however a short formula sheet will be available.

The final for this course is scheduled for **Tuesday, 17 March, between the lovely hours of 8:30 am to 11:30 am** and all of the material covered in the class is fair game. However, if a particular topic or problem gives the class as a whole more difficulty than others on the midterms, a portion of the final will emphasize that particular area.

[3] The omitted topics in mechanics are now included in the junior/senior elective course Physics 410, *Advanced Mechanics*, which is offered in alternate years. Also note, as I am sure previous members of this class will tell you, that I have decided to remove a section of the course that deals with the scientific text processing software, LaTeX, which is, in fact, what I am using to typeset these very words the week before school starts. For those of you who are interesting in learning LaTeX, our very own Sarah Marheine has offered to hold a division-wide tutorial; I encourage you to attend. I will also happily hold an extra session some time during the term if there is sufficient interest. In any case, you have a nice primer by way of your texts. I should conclude by saying that a knowledge of LaTeXis certainly useful, but I wanted to make room for other topics, time permitting.

**GRADING:** Your course grade will be determined from the homework you complete, the assigned presentations during conference days, and performance on the two midterms and final. The grade distribution is:

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|---|---|
| HW ASSIGNMENTS | 30% |
| CONFERENCE | 5% |
| MIDTERM | 20% |
| MIGHTY BIG PROBLEM WRITE UP | 10% |
| MIGHTY BIG PROBLEM ORAL | 5% |
| FINAL EXAM | 30% |
| TOTAL: | 100% |

I will use as default the standard 90-80-70-60 percent grading scale to determine boundaries between A-B-C-D-F; however, I reserve the right to lower the distribution based on class performance. The distribution will NOT be raised. If the class average is 93%, then there will be a lot of A's, and I'll be ecstatic. As an estimate, the median score will be used to set the rough location for a grade of C+/B-. Your grades will be updated routinely and posted on the course website, so you should have a good idea of where you are in the course at any time after the first midterm (at which time I will compute an estimated course grade for you based on the material in hand).

**HONOR SYSTEM:** Each student is expected to present only her or his own work on the hour examination and the final examination. In contrast, students are encouraged to work together on the assignments. Each student is expected to write up her or his own assignments, but working together to solve the problems can be a valuable learning aid. It is NOT acceptable to merely copy someone else's words or work.

I establish only two ground rules: (1) Working together will be most effective if all individuals contribute more or less equally to the group effort; you should be wary if you are always on the receiving end in such effort, for ultimately you will be expected to perform on your own. At the very least, once you have finished solving an exercise in a group, make sure that you could solve a similar exercise on your own. (2) Where substantial help has been received through conversation with another, I ask that you follow common scientific courtesy and acknowledge that help briefly in your submitted work.[4]

**ADA AND ANTIDISCRIMINATION STATEMENTS:** In compliance with the Americans With Disabilities Act (ADA), all qualified students enrolled in this course are entitled to reasonable accommodations. It is your responsibility to inform me of any special needs you may need before the end of the second week of classes. I also believe firmly in the right for each student to be respected, bo th by a student's peers and by his or her instructor. I am therefore strongly committed to ensuring that the antid iscrimination policy established at Lawrence will be honored in my class.

[4] For example, I would be remiss if I did not acknowledge the tremendous and gracious help I received from Professor Cook while preparing this syllabus, which is based on a template he supplied.

Physics 225 TENTATIVE SCHEDULE Winter Term, 2009

Abbreviations used in this schedule are C for conference session, H for hour examinations, L for lecture, P for problems due, N for no class *NCM* for *Notes for Computational Mechanics*, *CPSUP* for *Computation and Problem Solving in Undergraduate Physics*,and *LG* for the *Local Guide*. Unless otherwise noted, assignments are due at the start of class on the indicated days.

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|---|---|---|
| 05 Jan | M | L: Mathematics Review<br>(*NCM* pp 101–121) |
| 07 Jan | W | L: Kinematics, Dynamics, and Newton's Laws<br>(*NCM* pp 201–214) |
| 09 Jan | F | L: Momentum, Energy, & Rotation<br>(*NCM* 214–230) |

*************************** END OF WEEK 1 *************************

| | | |
|---|---|---|
| 12 Jan | M | C: Conference on Assignment 1 & Catch-up |
| 14 Jan | W | **P: ASSIGNMENT 1 DUE**<br>L: Forces and Simple Applications without ODEs<br>(*NCM* pp 301-324) |
| 16 Jan | F | N: NO CLASS; Work with IDL and TGIF in the CPL<br>(*CPSUP* Chapter 2, Sections 1-12, and Appendix B) |

*************************** END OF WEEK 2 *************************

| | | |
|---|---|---|
| 19 Jan | M | N: NO CLASS; Martin Luther King, Jr. Day |
| 21 Jan | W | L: Equations of Motion<br>(*NCM* pp 401–414; *CPSUP* Sections 9.1.1, 9.1.4, 9.1.6, and 9.1.7) |
| 22 Jan | R | **P: ASSIGNMENT 2 DUE** |
| 23 Jan | F | L: Constant, Time-Dependent, and 1D Position-Dependent Forces<br>(*NCM* pp 415–432) |

*************************** END OF WEEK 3 *************************

| | | |
|---|---|---|
| 26 Jan | M | L: Potential Energy & Velocity Dependent Forces<br>(*NCM* pp 433–445) |
| 28 Jan | W | L: Oscillations<br>(*NCM* pp 446–464) |
| 30 Jan | F | C: Conference on Assignment 3 & Cacth-up |
| 31 Jan | Sa | **P: ASSIGNMENT 3 DUE by noon** |

*************************** END OF WEEK 4 *************************

| | | |
|---|---|---|
| 02 Feb | M | **H: HOUR EXAMINATION #1 (covering Assignments 1, 2, 3)** |
| 04 Feb | W | L: Motion in a Plane, Central Forces, Effective Potentials, Orbital Equation |
| | | (*NCM* pp 501-515) |
| 06 Feb | F | L: Planets, Satellites, Comets |
| | | (*NCM* pp 516–536) |

*************************** END OF WEEK 5 *************************

| | | |
|---|---|---|
| 09 Feb | M | L: More Celestial Mechanics |
| 11 Feb | W | C: Conference on Assignment 4 & Catch-up |
| 13 Feb | F | NO CLASS; MIDTERM READING PERIOD |

*************************** END OF WEEK 6 *************************

| | | |
|---|---|---|
| 16 Feb | M | **P: ASSIGNMENT 4 DUE** |
| | | L: Using MAPLE to Solve ODEs |
| | | (*CPSUP* Section 9.4) |
| | | (*CPSUP* Chapter 5, Sections 1–8, 11, and 12) |
| 18 Feb | W | N: NO CLASS; Work with MAPLE in the CPL |
| 20 Feb | F | L: Algorithms for Solving ODEs Numerically |
| | | (*CPSUP* Section 9.6) |

*************************** END OF WEEK 7 *************************

| | | |
|---|---|---|
| 23 Feb | M | **P: ASSIGNMENT 5 DUE** |
| | | N: NO CLASS; Work with IDL in the CPL |
| | | (*CPSUP* Section 9.7) |
| 25 Feb | W | L: Using IDL to Solve ODEs Numerically |
| | | (*CPSUP* Section 9.7) |
| 27 Feb | F | C: Conference on Assignment 6 & Catch-up |
| 28 Feb | Sa | **P: ASSIGNMENT 6 DUE** |

*************************** END OF WEEK 8 *************************

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|---|---|---|
| 02 Mar | M | L: Using MAPLE to Evaluate Integrals Symbolically |
| | | (*CPSUP* Sections 11.1 and 11.3) |
| 04 Mar | W | L: Algorithms for Evaluating Integrals Numerically |
| | | (*CPSUP* Section 11.5) |
| 06 Mar | F | L: Using IDL to Evaluate Integrals Numerically |
| | | (*CPSUP* Section 11.6) |
| 07 Mar | Sa | **P: ASSIGNMENT 7 DUE by 5:00 pm** |

*************************** END OF WEEK 9 *************************

| | | |
|---|---|---|
| 9 Mar | M | N: NO CLASS; Work on MBP |
| 11 Mar | W | C: MBP Presentations |
| 13 Mar | F | C: MBP Presentations |
| 14 Mar | Sa | **P: MBP WRITE-UP DUE by noon** |

*************************** END OF WEEK 10 ************************

**ASSIGNMENT 1** (due Wednesday, 14 January 2008)

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|---|---|
| Reading: | *NCM* Chapters 1-2 |
| | *LG*, especially Sections 1, 2, 3, 4, 5, and 18. |
| Suggested: | Shankar, Chapter 1, pp 1–29 |
| | Shankar, Chapter 7, pp 149–159 |
| Problems 1:[5] | SP1.1, SP.2 |
| | P1.7, P1.15, P1.31, P1.34, P2.4, P2.15, P2.19, P2.21, P2.29, P2.34 |
| Problems 2: | P1.4, P1.5, P1.7, P1.17, P1.20, P1.30, P2.6, P2.17, P2.20, P2.23, P2.33 |

**SP1.1.** Work your way through the "whirlwind tutorial" contained in Section 2 of the *Local Guide*. This tutorial helps you determine your user name and initial password, leads you through the login process, helps you change your password, and then guides you through a few exercises designed to familiarize you with the rudimentary capabilities of software available in the CPL. There is nothing to *hand in* for this "problem", but it must be done by the due date for this assignment, since subsequent assignments will assume that you are already familiar with the contents of this tutorial. Note also the schedule when a consultant will be in the CPL to offer assistance; the first two or three times you log in, it is useful to have a more experienced individual nearby.

**SP1.2.** Sign on to the class's Moodle site and take the mechanics assessment quiz. Your results will not be graded, but you must complete this quiz no later than MIDNIGHT, WEDNESDAY of the first week of classes. Note Well: bf IN ORDER TO PASS THIS COURSE, YOU MUST COMPLETE THIS QUIZ BEFORE THE DEADLINE. The quiz is an assessment tool that we as a department are using in order to better measure baseline concepts you learned in Physics 150. Once you begin, you will have 30 minutes to complete 30 multiple choice questions. At the end of the quiz, you can see your score. The login password for the moodle site is `fortranrulz`. (and yes: yes it does.)

[5] Problems whose number is preceded with the characters SP will be presented on the assignment sheet itself; SP1.1, for example, is just below the listing of the problems. Problems whose number is preceded with the character P are in *NCM*, either at the end of the appropriate section or at the end of the appropriate chapter.

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**ASSIGNMENT 2** (due AT NOON, Thursday, 22 January 2008)

*Throughout the assignments, the notation CPSUP 2/3, for example, means that the edit at bullet 3 on page 2 of the CPSUP errata sheet should be noted if applicable.*

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|---|---|
| Reading: | *NCM* Chapter 3<br>*CPSUP* Chapter 2, especially Sections 1–3, 5, 7–12 (*CPSUP* 2/3 and 3/1, which goes at the end of Section 2.3.1. Please also read 'massaged' for 'messaged' in this edit.)<br>*CPSUP* Appendix B |

*N. B.*: The example statements to the computer included in the text are intended to be executed. Don't fail to enter those statements, observe their effect, and perhaps explore the behavior of slightly modified statements before addressing the problems to be handed in.

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| Problems 1: | P3.8 [omit part (c)], P3.13, P3.15, P3.19<br>[6]2.8, 2.14, 2.18, 2.24 (*CPSUP* 2/5), 2.30, B.2 |
| Problems 2: | P3.3, P3.5, P3.17<br>2.12, 2.21, 2.25, P3.8 [part(c)] |

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**ASSIGNMENT 3** (due at NOON, Saturday, 31 January 2008)

*N. B.*: This is a particularly long assignment. Do not postpone starting it. Problems occur at the end of the sections containing the necessary background, so you can tell easily from your readings and from the content of class sessions when you should be able to do each problem. Do *not* wait until two days before the assignment is due to start the problems. Such behavior will inevitably lead to frustration, a couple of long evenings, and an incomplete assignment.

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|---|---|
| Reading: | *NCM* Chapter 4 and, for review, *NCM* Section 1.4<br>*CPSUP* Sections 9.1.1, 9.1.4, 9.1.6, and 9.1.7 |
| Suggested: | Shankar, Chapter 7, pp 159–162, 167–181<br>Shankar, Chapter 3, pp 51–54; Chapter 10, pp 305–312<br>Shankar, Chapter 8, pp 205–219; Chapter 9, pp 255–266 |
| Problems 1: | P4.13, P4.27, P4.28, P4.29, P4.30, P4.41, P4.44, P4.50, P4.51, P4.60, P4.63, P4.70 |
| Problems 2: | P4.26, P4.38, P4.39, P4.40, P4.42, P4.43, P4.49, P4.56, P4.64, P4.65, P4.71 |

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[6] Problems whose number is preceded by no special character are in the indicated chapter of *CPSUP*; The problem numbered 2.8, for example, is at the end of Chapter 2 in *CPSUP*; Problem B.2 is at the end of Appendix B in *CPSUP*.

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**ASSIGNMENT 4** (due Monday, 16 February 2008)

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| Reading: | *CPSUP* Appendix A (*CPSUP* 7/6,7,8) |
| | *NCM* Chapter 5 |
| Problems 1: | P5.1, P5.5, P5.6, P5.12, P5.13, |
| | P5.17, P5.30, P5.32, P5.34 |
| | SP5.1 |
| Problems 2: | P5.3, P5.15, P5.20, P5.22, P5.23, P5.28, P5.38 |

**SP4.1.** Least Energy Orbit. Using the principles discussed in class, describe in detail the least energy orbit trajectories for an interplanetary probe launched from the Earth to Mercury. In other words, find the minimum $\Delta v$ for launch, the time of flight, and the speed on arrival relative to the Sun and to Mercury. Does the spacecraft need to adjust its velocity to obtain a low altitude circular orbit about Mercury? If so, by how much?

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**ASSIGNMENT 5** (due Monday, 23 February 2008)

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| Reading: | *CPSUP* Sections 5.1–5.8 (especially 5.8.9), 5.11, 5.12 (*CPSUP* 3/2, 4/1,2,3,4,5) |
| | *CPSUP* Section 9.4 |
| Problems 1: | 5.5; 5.12 (*CPSUP* 5/2); SP5.1; SP5.2a,b; 9.2 |
| Problems 2: | 5.7, 5.16, 9.7 |

**SP5.1.** Using MAPLE, obtain graphs of the potential in P4.27 and the associated force. Note that the expression for the potential is simpler to plot if you plot not $U(x)$ versus $x$ but rather $V(x)/V_0$ versus $\overline{x} = x/a$, i.e., rewrite the function in the form

$$U(x) = -\frac{U_0 a^2(a^2 + x^2)}{8a^4 + x^4} \qquad \longrightarrow \qquad \frac{U(x)}{U_0} = -\frac{1 + \overline{x}^2}{8 + \overline{x}^4}$$

You should be able to find a similar dimensionless version of the expression giving the force. Be sure your submitted solution explains fully how you used MAPLE to generate the desired graphs. (*Note*: There are numerous ways to obtain hard copy from MAPLE. Probably the easiest involves working in the GUI interface (started with the command `xmaple`) because, in that interface, you can

- Enter commands at the MAPLE prompt.
- Hide all your false starts by removing commands that don't work. (Select the erroneous paragraph by clicking ML on the opening square bracket that delineates it and then select 'Delete Paragraph' from the EDIT menu.)
- Enter textual descriptions as described in *CPSUP* Section 5.14.
- Display graphs inline rather than in a separate window by executing the command `plotsetup(inline)` to MAPLE.
- Print the entire contents of the window by selecting 'Print' from the FILE menu. (In the *Printer Setup* window that comes up, select 'Print Command' and make sure the command in the text entry box is `lp`, select 8.5"×11" paper size, and select 'Portrait' for the page layout before clicking ML on the 'Print' button.)

The contents of the MAPLE window can also be exported to a file in several formats, some of which can be processed with LaTeX. Those issues are discussed in Section 5.13, which you should feel free to explore even though it is not officially assigned.

**SP5.2.** Using MAPLE, find complete (symbolic) solutions to each of the following problems, and use MAPLE also to verify that the solutions you obtain actually *do* satisfy the original ODE and initial conditions.

a. $\dfrac{d^2x}{dt^2} = a$, $x(0) = x_0$, $v(0) = v_0$, where $a$ is constant, i.e., find position as a function of time for a particle moving under the action of a constant force and launched with arbitrary initial conditions.

b. $m\,\dfrac{d^2x}{dt^2} = -eE_0\cos(\omega t + \theta)$, $x(0) = x_0$, $v(0) = v_0$, i.e., find position as a function of time for a charged particle moving under the action of a sinusoidal force and launched with arbitrary initial conditions.

c. $m\,\dfrac{d^2x}{dt^2} = -mg + b\left(\dfrac{dx}{dt}\right)^2$, $x(0) = 0$, $v(0) = 0$, i.e., find position as a function of time for a particle released from rest at the origin and allowed to fall freely under the action of gravity and a viscous retarding force proportional to the *square* of the speed.

d. the differential equations for $x(t)$ and $z(t)$ in P4.50.

Be sure your submitted solution explains fully how you used MAPLE to generate the desired solutions. (*Note*: See the note in Problem SP5.1.)

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**ASSIGNMENT 6** (due at NOON, Saturday 28 February 2008)

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| Reading: | *CPSUP* Sections 9.6 and 9.7 |
| Suggested: | Feynman, Volume 1, Lecture 9 |
| Problems 1:[7] | SP6.1, SP6.3, 9.18, any two—your choice—of (9.17, 9.19, 9.23) |
| Problems 2: | SP6.4, SP6.5, the other of (9.17, 9.19, 9.23), 9.27 |

**SP6.1** (See also P5.12) A particle of mass $m$ moves under the action of a central force whose potential is

$$U(\rho) = K\rho^4, \qquad K > 0$$

Find the equations of motion for a planet moving under the influence of this force, cast them in suitable dimensionless form, and use numerical methods to explore both circular orbits (as verification of your numerical procedures) and orbits having other initial conditions. Try approaching the problem both in Cartesian coordinates and in polar coordinates. Write several paragraphs describing and presenting evidence for your discoveries.

**SP6.3.** In P4.27, we found that the potential

$$U(x) = -U_0\frac{a^2(a^2+x^2)}{8a^4+x^4} \qquad \text{or} \qquad \frac{U(x)}{U_0} = -\frac{1+\overline{x}^2}{8+\overline{x}^4}$$

where $\overline{x} = x/a$ exhibited a variety of motions (bound motion in either the left or the right well when $-\frac{1}{4} \le E/V_0 \le -\frac{1}{8}$; bound motion over the whole well when $-\frac{1}{8} \le E/V_0 \le 0$; unbound motion when $E/V_0 > 0$). Find the equation of motion for this system, cast it in a suitable dimensionless form, and then explore each of the different types of motion possible. Be sure to look both at graphs of $x$ and $v = dx/dt$ versus $t$ *and*—especially for the periodic motions—at the phase plane plots, $v$ versus $x$—all in suitable dimensionless units. It might be interesting to examine period as a function of energy for this oscillator. Write several paragraphs describing and presenting evidence for your discoveries. *Hint*: Perhaps a suitable dimensionless time would be $\bar{t} = \omega_0 t$, where $\omega_0$ is the frequency of small amplitude oscillations when the particle is confined to one side of the well.

**SP6.4.** A 1000 kg mass is dropped on a platform from a height of 10 m (acquiring a velocity of 14.0 m/s as it hits the platform). The spring is characterized by a constant $k = 49000$ N/m and the mass strikes the platform a distance $x_0 = 0.2$ m above the desired equilibrium point. The undamped frequency of oscillation is $\omega_0 = 7.0$ rad/s and the ratio $\gamma = b/2m$ for *critical* damping is also 7.0 rad/s. The equation of motion is

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$$

[7] Problems whose number is preceded by no special character are in the indicated chapter of *CPSUP*; the problem numbered 9.17, for example, is at the end of Chapter 9 in *CPSUP*.

to be solved subject to the requirements $x(0) = x_0 = 0.2$ m and $v(0) = -14.0$ m/s. (a) Show that this problem can be recast in the dimensionless form

$$\frac{d^2\overline{x}}{d\overline{t}^2} + 2q\,\frac{d\overline{x}}{d\overline{t}} + \overline{x} = 0 \qquad ; \qquad \overline{x}_{\overline{t}=0} = 1 \qquad ; \qquad \left.\frac{d\overline{x}}{d\overline{t}}\right|_{\overline{t}=0} = -10$$

where $\overline{x} = x/x_0$, $\overline{t} = \omega_0 t$, and $q = b/2\omega_0 m = \gamma/\omega_0$. (Note that $q = 1$ for the case of *critical* damping.) (b) Explore the solution to this problem numerically for several values of $q$, including values both larger and smaller than 1. In particular, note the overshoot when $q = 1$ and search for the value of $q$ at which overshoot first disappears. What is the value of $b$ at that point?

**SP6.5.** One type of child's swing is hung with elastic ropes. Suppose the ropes are long enough so that the child/swing can be represented by a point mass at the end of a spring, assume the spring obeys Hooke's law and has an unstretched length $a$, and let the motion of the mass be confined to a single vertical plane. Show that, in the coordinate system illustrated, the equations of motion are

$$m\frac{d^2x}{dt^2} = -kx + \frac{kax}{\sqrt{x^2+y^2}} \qquad ; \qquad m\frac{d^2y}{dt^2} = mg - ky + \frac{kay}{\sqrt{x^2+y^2}}$$

Then, introducing $\omega_0^2 = k/m$, $\overline{t} = \omega_0 t$, $\overline{x} = x/a$, and $\overline{y} = y/a$, cast the equations in dimensionless form. After the suggested rescalings, only one parameter—$g/a\omega_0^2$—remains [and *this* parameter is the (square of the) ratio of the frequency $\sqrt{g/a}$ of a simple pendulum of length $a$ to the frequency $\sqrt{k/m}$ of a mass $m$ bobbing on a spring of stiffness $k$]. Explore the motions for several values of this one parameter, including values larger than, equal to, and smaller than 1. Write several paragraphs describing and presenting evidence for your discoveries.

**SP6.6.** Use IDL to study the motion of a mass sliding on a horizontal surface under the action of a force from a spring and the force of dry friction. The equation of motion is

$$m\frac{d^2x}{dt^2} = -kx - \mu\, mg\, \mathrm{sgn}\left(\frac{dx}{dt}\right)$$

where $\mathrm{sgn}(z)$ is the signum function, $+1$ when $z > 0$, $-1$ when $z < 0$, and 0 when $z = 0$. Be particularly careful to lay out a criterion for determining when the motion has stopped. Write several paragraphs describing and presenting evidence for your discoveries.

**SP6.7.** Consider a damped pendulum but add a sinusoidal driving force. In dimensionless form, the equation of motion is

$$\frac{d^2\theta}{dt^2} + \Gamma\frac{d\theta}{dt} + \sin\theta = A\cos(\Omega t)$$

where the three parameters are $\Gamma$, which is a dimensionless measure of the damping; $A$, which is a dimensionless measure of the amplitude of the driving force, and $\Omega$, which is the ratio of the frequency of the driving force to the small amplitude natural frequency of the undamped pendulum. This equation is conveniently recast in terms of the variables $\theta$, $\omega = d\theta/dt$, and $\phi = \Omega t$, and we find that

$$\frac{d\theta}{dt} = \omega \qquad ; \qquad \frac{d\omega}{dt} = -\Gamma\omega - \sin\theta + A\cos\phi \qquad ; \qquad \frac{d\phi}{dt} = \Omega$$

This system exhibits an enormous range of behaviors, depending on the relationships among the three parameters. You should look particularly at the way the behavior changes with change in the amplitude $A$ of the driving force. Thus, at least at the start—you may explore other values later if you wish—take $\Gamma = 0.5$ and $\Omega = \frac{2}{3}$. Study the *steady state*—i.e., wait long enough for any initial transients in the motion to have died out before plotting graphs—behavior of this system for various driving amplitudes, including

$A = 0.5, 1.07, 1.35, 1.45, 1.5$. Look particularly at the phase plane plots $\omega$ versus $\theta$. Write several paragraphs describing and presenting evidence for your discoveries.

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**ASSIGNMENT 7** (due at NOON, Saturday, 7 March 2007)

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| Reading: | *CPSUP* Chapter 11, Sections 1, 3, 5, and 6 (*CPSUP* 6/1,2) |
| Problems 1:[8] | 11.4, 11.7, 11.10, 11.18, 11.24, 11.27 |
| Problems 2: | 11.2, 11.5, 11.6, 11.22, 11.25, 11.29 |

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[8] Problems whose number is preceded by no special character are in the indicated chapter of *CPSUP*; the problem numbered 11.14, for example, is at the end of Chapter 11 in *CPSUP*.